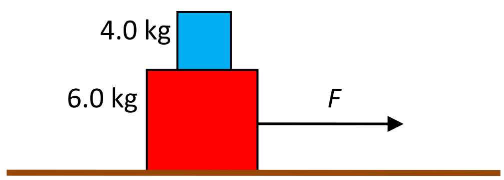
Teacher notes
Topic A

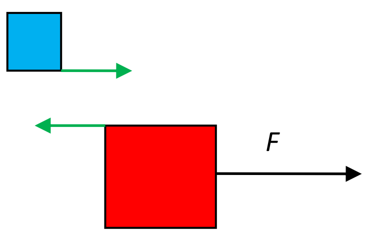
A block of mass 4.0 kg rests on top of another block of mass 6.0 kg. The coefficients of friction between the blocks are 0.60 for the static and 0.40 for the kinetic. There is no friction between the heavier block and the horizontal ground. Take $g = 10\,\mathrm{m\,s^{-2}}$.



A horizontal force *F* is applied to the heavier block. Determine the largest magnitude of *F* so that both blocks move together without sliding on each other.

Approach

What force pushes the lighter block? Imagine doing the experiment. If you push the lower block with a very large force, the lighter block will slide backwards. So a frictional force will develop to oppose this sliding. If *F* is not too large, both bodies will move together and the force accelerating the lighter block forward is the frictional force between the blocks. It is directed to the right. By Newton's third law, an equal force is exerted on the larger block in the opposite direction.



Since we want the largest force *F* we need to use the largest possible frictional force that can develop between the blocks and that is given by $f = \mu_s N = 0.60 \times 40 = 24\,\mathrm{N}$. The acceleration of the lighter block is then $a = \dfrac{24}{4.0} = 6.0\,\mathrm{m\,s^{-2}}$. This must also be the acceleration of the second block. Thinking of the blocks as one of mass 10 kg, the net force is just *F* and so we find $F = Ma = 10 \times 6.0 = 60\,\mathrm{N}$.

We could also argue as follows: the lower block exerts a frictional force of 24 N on the upper block. By Newton's third law then, there is a force of 24 N acting on the lower block to the left. The net horizontal force on the lower block is then $F-24$ and since the acceleration is $6.0\,\text{ms}^{-2}$ we have $F-24=ma=6.0\times6.0=36$, i.e. $F-24=36$ giving again $F=60\,\text{N}$.

For the same data as above imagine now that F = 50 N. What is the frictional force pushing the lighter block forward now?

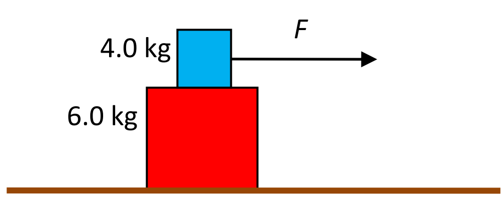
It is important to realize right away that it is not 24 N. The formula $f=\mu_s R$ gives the **maximum** possible force that can develop between the two blocks. It does not give the frictional force in general. In this case, we know that the two bodies move together (because 50 N < 60 N) so thinking of them as one body we find $F=Ma\Rightarrow 50=10a\Rightarrow a=5.0\,\text{ms}^{-2}$. The net force on the lighter block is the frictional force and so $f=ma=4.0\times5.0=20\,\text{N}$.

Now imagine that $F=64\,\text{N}$. What is each body's acceleration?

This force is larger than the 60 N we found before and so we know there will be sliding and so the two blocks will have different accelerations. With sliding we know right away that the frictional force is always given by $f=\mu_k R=0.40\times40=16\,\text{N}$. The acceleration of the lighter block is $a=\dfrac{16}{4.0}=4.0\,\text{ms}^{-2}$. The net force on the larger block is $F-16=64-16=48\,\text{N}$ and so the acceleration of the larger block is $a=\dfrac{48}{6.0}=8.0\,\text{ms}^{-2}$.

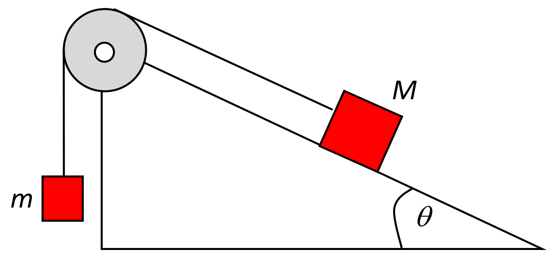
Note that we cannot think of the two bodies as one here, because there is sliding and so the two bodies move with different accelerations.

How does this problem change when the force F acts on the top block? What is the maximum force F for the blocks not to slide on each other?



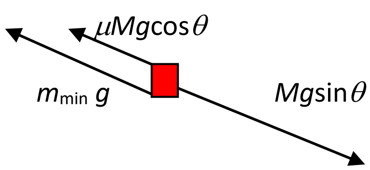
The maximum frictional force is again 24 N and this is the force accelerating the lower block. The acceleration is then $a=\dfrac{24}{6.0}=4.0\,\text{ms}^{-2}$. The net force on the two blocks taken as one is F and so $F=10\times4.0=40\,\text{N}$.

A block of mass *M* rests on a rough inclined plane. The coefficient of static friction between the block and the incline is 1. The plane makes an angle $\theta = 60^{\circ}$ with the horizontal. The block is connected to a hanging block of mass *m* through a string that goes over a pulley.



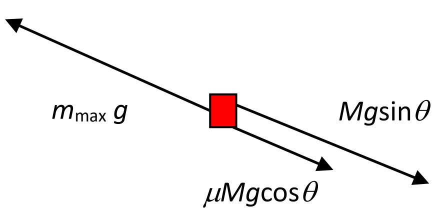
What is the ratio of the maximum to the minimum value of *m* for which we have equilibrium?

When the hanging mass has its minimum value, m_{min}, the equilibrium condition is (make a free body diagram)



$m_{min}g + \mu Mg\cos\theta = Mg\sin\theta$ or $m_{min} = M(\sin\theta - \cos\theta)$ since μ = 1.

When it has its maximum value, m_{max}, the condition is (make a free body diagram)



$m_{max}g = Mg\sin\theta + \mu Mg\cos\theta$ or $m_{max} = M(\sin\theta + \cos\theta)$.

Taking ratios

$$\frac{m_{max}}{m_{min}} = \frac{M(\sin\theta + \cos\theta)}{M(\sin\theta - \cos\theta)} = \frac{\sin 60^{\circ} + \cos 60^{\circ}}{\sin 60^{\circ} - \cos 60^{\circ}} = 3.73$$

Notice that since $\mu = 1$, the block can be in equilibrium on the incline for angles up to $\theta = \arctan 1 = 45^\circ$ without any support from the hanging mass. Here we have equilibrium at a larger angle because of the hanging mass. If M = 10 kg, $m_{\text{max}} = 10(\sin 60^\circ + \cos 60^\circ) = 13.7\text{ kg}$ and $m_{\text{min}} = 10(\sin 60^\circ - \cos 60^\circ) = 3.7\text{ kg}$.